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Static chaos in spin glasses: the case of quenched disorder perturbations

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Abstract. We study the chaotic nature of spin glasses against perturbations of the realization of the quenched disorder. This type of perturbation modifies the energy landscape of the system without adding extensive energy. We exactly solve the mean-field case, which displays a very similar chaos to that observed under magnetic field perturbations, and discuss the possible extension of these results to the case of short-ranged models. It appears that dimension four plays the role of a specific critical dimension where mean-field theory is valid. We present numerical simulation results which support our main conclusions.

1. Introduction

A long debated problem in spin glass theory concerns the correct description of the statics of the low-temperature phase [1]. There is wide consensus on the fact that the mean-field theory is well understood in its essentials, while the nature of the equilibrium states for short-ranged models is still a controversial subject. Two competing pictures or approaches have been proposed: the mean-field picture and the droplet model. The mean-field theory has been shown to be enormously complex if used to provide a comprehensive approach to understand short-range models. Consequently, the search for different approaches like droplet models [2] has been encouraged. These models, being phenomenological, try to capture the main aspects underlying the equilibrium and non-equilibrium properties of short-ranged models. Unfortunately, the mean-field way and these phenomenological approaches are far from being complementary and much effort has been devoted in recent years in discerning which is the correct picture. Numerical simulations have played a prominent role in this task even though the main question still remains unsolved. The main problem relies on the large amount of computer time needed in order to reach equilibrium.

Despite the fact that both pictures are, in fact, contradictory in their essentials, there are, however, some common predictions in both approaches. Since it is very difficult to decide which is the correct picture, the strategy of searching for common features in both pictures can be useful to shed light on this controversy. Static chaos appears to be a good starting point for this programme. By static chaos we understand the sensitivity of the low-temperature phase of spin glasses to static perturbations, such as changes in temperature

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or magnetic field. Mean-field theory [3] and droplet models [2] predict that spin-glasses, in the most general case, are chaotic. In mean-field theory, the mechanism of chaos is due to the small free energy differences between the different equilibrium states. These are of order $O(1/N)$ and a small perturbation completely reshuffles the Boltzmann weights $w_\alpha \sim \exp(-N\beta f_\alpha)$ of the different equilibrium states (α and f_α stand for equilibrium state and its free energy respectively). In droplet models, the application of a perturbation causes a reorganization of the spin–spin correlations at long distances. In both pictures the system is very sensitive to the applied perturbations.

A nice example of chaos concerns the sensitivity of spin-glasses to magnetic field perturbations [3,4]. The chaos exponent (to be defined in the next section) for this type of perturbation has been computed in mean-field theory [3] and numerically measured in short-ranged models [4]. Surprisingly, this chaos exponent does not depend on the dimensionality of the system [4]. Even though we do not know of a theoretical derivation of this result, it appears to be sound enough to be considered. Droplet models can give an explanation for this result under the assumption that 3 is the lower critical dimension in Ising spin glasses (also a long debated problem [5]). In the context of droplet models, the chaos exponent for magnetic field perturbations is related to the thermal exponent θ which measures the free energy cost of the droplet excitations. The result $\theta = (d-3)/2$ implies that the chaos exponent is 2/3 and does not depend on the dimension. This has to be compared to the known results, $\theta = -1$ (exact) in $d = 1$ [6], $\theta \simeq -0.48$ in $d = 2$ [7] and the exact result for the chaos exponent (2/3) in the Gaussian approximation to mean-field theory [3]. Apparently the simple expression previously reported for θ correctly matches the small d regime in the infinite dimension result.

Regarding other types of perturbation, the situation is more difficult. For instance in the case of temperature changes, recent results by Franz and Nifle [8] show that there is chaos even though it is expected to be small (as a first computation by Kondor [3] suggested). In the case of short-range models analytical computations by Kondor and Vegso [9] show that chaos exponents should depend on the dimension below six dimensions. As in the SK model, numerical results in four dimensions [10] show that the chaos in the temperature is also very small.

The perturbations previously commented upon share the common property that they add energy to the system. This work is devoted to the study of a perturbation which does not add extensive energy to the system. In particular we will study chaoticity with respect to changes in the realization of the quenched disorder. Because of the self-averaging property we expect that changes in the realization of the disorder (keeping the form of the disorder distribution) shoud not add extensive energy to the system. Therefore chaoticity appears because of a complete reshuffling of the free energies of the configurations. We will show that the system displays chaos very similar to that shown in magnetic field perturbations. Criticality of chaos against perturbations of the quenched disorder has been studied by other groups [11]. The perturbation in which we are interested differs from others by the fact that we change the sample realization without moving the system to a new point in the phase diagram.

The paper is organized as follows. The next section defines the model. Section 3 is devoted to the study of chaos in mean-field models. Section 4 presents the scaling approach we have used to obtain the chaos exponents and shows the numerical results. Finally we present our conclusions in section 5.

2. The model

We consider the model described by the Hamiltonian

$$H[\sigma] = -\sum_{(i,j)} J_{ij}\sigma_i\sigma_j - h\sum_i \sigma_i. \tag{1}$$

The couplings in (1) are symmetrically distributed random variables with zero mean and finite variance (in the mean-field case the variance goes like $1/N$ where N is the number of spins). The perturbation we consider consists in randomly changing the sign of a fraction r of the couplings, i.e. for each coupling we change its sign with probability r. On average, a total number Nr of the couplings J_{ij} are changed to $-J_{ij}$. In this way we keep the new configuration of Js in the same ensemble of disorder realizations without moving the system in the phase diagram. This is different from other types of perturbation in which, for instance (see [11]), the J_{ij} are changed by a small amount $\delta \cdot x_{ij}$ where x_{ij} is a random number and δ is small. In this case the variance of the distribution is increased (it grows proportionally with δ^2) and we add energy to the system. In what follows we will consider, for simplicity, the case of zero magnetic field.

Denoting by R the set of couplings which change sign, then we can write the perturbed Hamiltonian as

$$H_r[\sigma] = -\sum_{(i,j)} J_{ij}\sigma_i\sigma_j + 2\sum_{(i,j)\in R} J_{ij}\sigma_i\sigma_j. \tag{2}$$

The sum runs over nearest neighbours in a lattice of dimension d. The mean-field case can be obtained in several ways. In the infinite-range model or SK model, all the spins interact with one another. Alternatively, one can consider the finite connectivity random lattices (with fixed or average number of neighbours [12]).

Once we have defined the perturbation we construct a full Hamiltonian $H_{12}[\sigma, \tau]$ defined in a space of two sets of variables $\{\sigma_i, \tau_i; i = 1, \ldots, N\}$. The Hamiltonian H_{12} is the sum of the unperturbed Hamiltonian $H[\sigma]$ plus the perturbed Hamiltonian $H_r[\tau]$:

$$H_{12}[\sigma, \tau] = H[\sigma] + H_r[\tau]. \tag{3}$$

We define the usual spin-glass correlation functions

$$G(x) = \overline{\langle \sigma_0\tau_0\sigma_x\tau_x \rangle} \tag{4}$$

where $\overline{(\ldots)}$ means average over the quenched disorder and $\langle \ldots \rangle$ corresponds to the thermal average over the full Hamiltonian H_{12}. The degree of coherence of the two systems is measured by the overlap function

$$P(q) = \overline{\left\langle \delta\left(q - \frac{1}{N}\sum_i \sigma_i\tau_i\right)\right\rangle}. \tag{5}$$

At large distances $G(x)$ behaves like

$$G(x) \sim \exp(-x/\xi(r)) \tag{6}$$

where $\xi(r)$ is the chaos correlation length, which is finite for a finite perturbation r (we identify the perturbation with the fraction r of changed couplings). The chaos correlation length diverges when $r \to 0$ if the unperturbed system stays in the spin-glass phase, including the critical point. This is because in the limit $r \to 0$, $G(x)$ converges to the usual spin-glass correlation function which has infrared singularities due to the existence of zero modes. The chaos correlation length diverges:

$$\xi(r) \sim r^{-\lambda}. \tag{7}$$

This equation defines the chaos exponent λ which can be exactly computed in some particular cases. We expect that static chaos is absent when the system stays at a temperature above the spin-glass transition (the paramagnetic phase) because in this case $\xi(r = 0)$ is finite. Furthermore this is true only if $\xi(r)$ smoothly converges to the finite correlation length at that temperature when $r \to 0$. This is the case in mean-field theory but should not necessarily be true in finite dimensions [13]. The exponent λ can also depend, in principle, on the temperature. We will show in mean-field theory that λ is constant in the low-T phase. At the critical point we face two posibilities. There can be a new chaos exponent independent of the usual critical exponents, as suggested in [14]. But it could be that the chaos exponent λ depends only on the critical exponents. A relation between the chaos exponent and the critical exponents in finite dimensions can be obtained as follows: when a fraction r of the couplings change, the system changes its energy in a quantity proportional to the square root of the total number of changed couplings, i.e. $(rL^d)^{1/2}$ (because the perturbed bonds have random sign). Also the energy at the critical point scales like $L^{(\alpha-1)/\nu}$. This yields

$$\xi \simeq r^{\nu/\alpha} \tag{8}$$

which gives $\lambda = -\nu/\alpha$. This value of λ is positive only if $\alpha \leqslant 0$ which is a general result in spin glasses (there is no divergence of the specific heat at the critical point). This expression gives the correct exponent in mean-field theory (we will see $\lambda = \frac{1}{2}$) and compatible results in four dimensions (see section 4).

3. Chaos in mean-field theory

Now we face the problem of computing the exponent λ in mean-field theory. We follow the standard procedures (see [4] for details) and we apply the replica method to the full Hamiltonian H_{12} (3), where the random variables J_{ij} have $1/N$ variance:

$$-\beta f = \lim_{n\to 0} \frac{\log(\overline{Z_J^n})}{nN}. \tag{9}$$

Introducing Lagrange multipliers for the different order parameters one gets a saddle-point integral

$$\overline{Z_J^n} = \int \mathrm{d}P\,\mathrm{d}Q\,\mathrm{d}R\,\exp(-NA[PQR]) \tag{10}$$

with

$$A[PQR] = \frac{\beta^2}{2}\sum_{a<b}(P_{ab}^2 + Q_{ab}^2) + \frac{\beta^2}{2}\sum_{a,b}(R_{ab}^2) - \log(Tr_{\sigma\tau}\exp(L)) \tag{11}$$

where a, b denote replica indices which run from 1 to n, and

$$L[\sigma,\tau] = \beta^2\sum_{a<b}(Q_{ab}\sigma_a\sigma_b + P_{ab}\tau_a\tau_b) + \beta^2\sqrt{1-2r}\sum_{a,b}(R_{ab}\sigma_a\tau_b). \tag{12}$$

We note that the previous expression is not invariant with respect to the change $r \to 1-r$. In fact, redefining the matrix $R' = \mathrm{i}R$ (i stands for the imaginary unit) equation (12) is invariant, while the term $\sum_{a,b} R_{ab}^2$ in equation (11) changes sign. This means that when $r = 1$, the only solution to the equations of motion is $R = 0$ which means that pure states for the initial sample and the completely perturbed one are totally uncorrelated. In contrast, the symmetry $r \to 1-r$ is preserved in hypercubic finite dimensional lattices where there is a gauge symmetry between the configurations of the initial sample and the perturbed one.

For finite r values there is one stable solution to the equations of motion:

$$Q_{ab} = P_{ab} = Q_{ab}^{\mathrm{SK}} \qquad R_{ab} = 0 \tag{13}$$

where Q_{ab}^{SK} is the solution for the unperturbed system. The order parameter R_{ab} measures the degree of correlation equation (5) between the two systems via the relation

$$P(q) = \frac{1}{n^2}\sum_{a,b}\delta(q - R_{ab}). \tag{14}$$

The stability of the solution $R = 0$ means that there is chaos against coupling perturbations. This is indeed very similar to the case of chaos in a magnetic field and chaos against temperature changes. Now we can compute, in the Gaussian approximation, the correlation function $G(x)$ of equation (6). The computations can easily be done in Fourier space. We define

$$C(p) = \sum_x G(x)\mathrm{e}^{\mathrm{i}px}. \tag{15}$$

In order to find $C(p)$ we need to compute the spectrum of fluctuations in the direction R_{ab} around the stable solution (equation (13)). The full expression has been reported in [3]. This yields the singular behaviour of the correlation function in the spin-glass phase:

$$C(p) \sim p^{-4} \qquad p \to 0 \tag{16}$$

and at the critical temperature [4],

$$C(p) \sim p^{-2} \qquad p \to 0. \tag{17}$$

The chaos correlation length $\xi(r)$ is obtained from the minimum eigenvalue of the stability matrix:

$$H_{ab,cd} = \frac{\partial^2 A}{\partial R_{ab}\partial R_{cd}} = \beta^2\delta_{(ac)}\delta_{(bd)} - \beta^4(1-2r)(\langle\sigma_a\tau_b\sigma_c\tau_d\rangle - \langle\sigma_a\tau_b\rangle\langle\sigma_c\tau_d\rangle) \tag{18}$$

where the expected values $\langle\ldots\rangle$ stand for average over the effective action L. A computation similar to the first by Kondor [3] gives

$$\lambda_{\min} = 2\beta^2 r \tag{19}$$

and this yields

$$\xi(r) \sim \lambda_{\min}^{-1/2} \sim r^{-1/2}. \tag{20}$$

This result is valid at and below the critical point. We expect it to be valid in other mean-field models also such as, for instance, finite connectivity random lattices. In this case, where analytical calculations become much more involved, we expect to obtain the same results. This will be nicely corroborated by our numerical simulations in section 4.

4. Numerical results

In this section we will discuss our Monte Carlo simulations in order to test the results obtained in the previous sections for the chaos exponents. Furthermore, we will present simulations in four dimensions. Our results are compatible with the fact that the chaos exponents in finite dimensions are compatible with the mean-field ones.

We have simulated two types of mean-field model (the Sherrington–Kirkpatrick (SK) model [15] and the random finite connectivity lattice model [12]) and a four-dimensional (4D) Ising spin glass for which the existence of a finite T phase transition is well established [16]. Monte Carlo simulations implement the Metropolis algorithm (for the mean-field models) and the heat-bath algorithm (in the 4D case). Special attention has been paid in order to thermalize the samples.

4.1. The finite-size scaling approach

In order to measure the chaos exponents, we have performed a finite-size scaling analysis [4]. The idea is to compute the overlap between two copies of the system, one copy with an initial realization of the disorder, the other one with the perturbed realization. The overlap is defined as

$$q = \frac{1}{N}\sum_{i=1}^{N}\sigma_i\tau_i. \tag{21}$$

We define the *chaos parameter* $a(r)$:

$$a(r) = \frac{\overline{\langle\sigma_i\tau_i\rangle^2}_r}{\overline{\langle\sigma_i\tau_i\rangle^2}_{r=0}} \tag{22}$$

i.e. we normalize the correlation between the unperturbed and the perturbed system to the autocorrelation of the unperturbed system. In this way $a(0) = 1$ by definition. The system is chaotic if the quantity $a(r)$ jumps to 0 (in the thermodynamic limit) as soon as r is finite. This means that

$$\lim_{r\to 0}\lim_{N\to\infty} a(r) = 0 \qquad \text{while } a(0) = 1. \tag{23}$$

It is crucial to perform the limits in the order previously indicated. Since a is an adimensional quantity, we expect it to scale like

$$a \equiv f(L/\xi) \tag{24}$$

where ξ is the chaos correlation length of equation (6). In the mean-field case we find, for the spin-glass phase (using equations (16) and (20)),

$$a \equiv f(Nr^2) \tag{25}$$

and at the critical point we get (using equation (17))

$$a \equiv f(Nr^3). \tag{26}$$

In the case of short-range models we expect the general scaling behaviour

$$a \equiv f(rL^{1/\lambda}) \tag{27}$$

where, in the general case, f is a different function in each of the above equations. Since only one exponent (the chaos exponent) must be fitted, these scaling relations are highly predictive. As we will show in the next section, our numerical determination gives (below T_c) $\lambda = \frac{1}{2}$ with good precision in four dimensions. The exactness of this results ($\lambda = \frac{1}{2}$) suggests (comparing equations (25) and (27)) that $d_u = 4$ plays the role of a critical dimension where the chaos exponent coincides with the mean-field one. The situation is the same as in the case of magnetic field perturbations [4], where the value of this dimension only depends on the behaviour of the propagator $C(p)$ in the limit $p \to 0$. Then we expect the scaling functions $f(x)$ in equations (25) and (27) to coincide except by the presence of some logarithmic corrections. A numerical test of this prediction is also shown in the following subsections.

4.2. Numerical results in mean-field models

The SK model is defined by the following Hamiltonian:

$$H = -\sum_{i<j} J_{ij}\sigma_i\sigma_j \tag{28}$$

with the J_{ij} distributed according to the function $p(J_{ij})$. In the thermodynamic limit, the only relevant feature of the $p(J_{ij})$ is its variance (we restrict to distributions with zero mean). To speed up the numerical computations we have taken a binary distribution of couplings, i.e. the Js can take the values $\pm\frac{1}{\sqrt{N}}$ with equal probability.

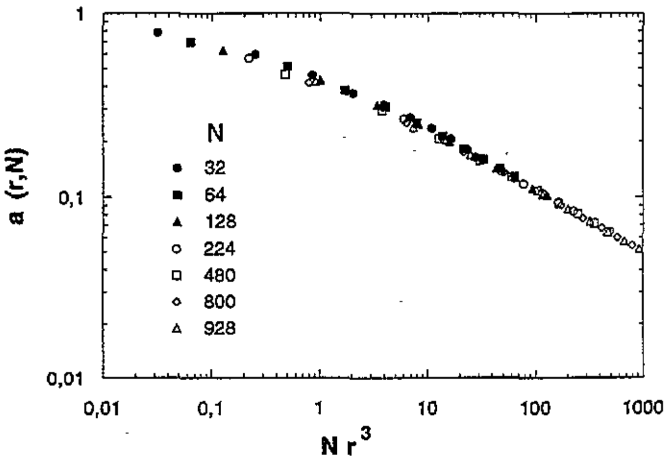

Figure 1. Chaos in the SK model at the critical point $T_c = 1$.

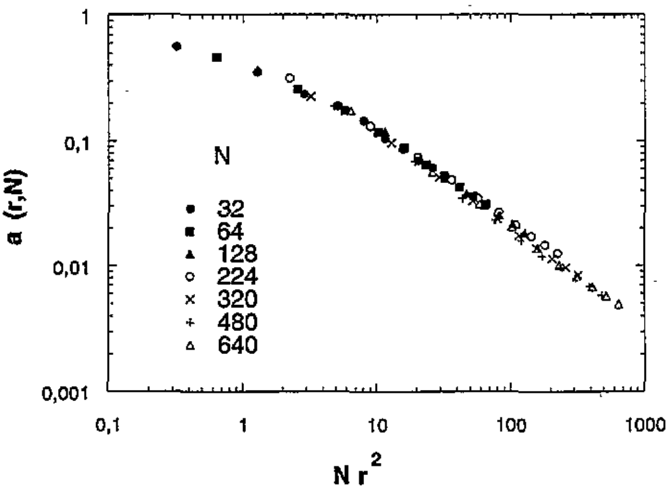

Figure 2. Chaos in the SK model at $T = 0.7$ in the spin-glass phase.

We have simulated the SK model at the critical temperature $T = 1$ and below the critical temperature. We have computed the chaos parameter a for different values of r (r runs from 0 to 1). Simulations were done for lattice sizes ranging from $N = 32$ to $N = 1000$.

Figures 1 and 2 show the scaling laws (25) and (26) at the critical point $T = 1$ and below the critical point $T = 0.7$ respectively. The data do nicely fit the predictions. We have also simulated the random finite-connectivity lattice model (FC model). In this model each point of the lattice is connected (on average) to a finite number c of neighbours†. In this case the FC model is defined by

$$H = -\sum_{i<j} J_{ij}\sigma_i\sigma_j \tag{29}$$

where the J_{ij} are distributed according to

$$\mathcal{P}(J_{ij}) = \frac{c}{N}p(J_{ij}) + \left(1 - \frac{c}{N}\right)\delta(J_{ij}) \tag{30}$$

and $p(J_{ij})$ is given by

$$p(J_{ij}) = \tfrac{1}{2}\delta(J_{ij} - 1) + \tfrac{1}{2}\delta(J_{ij} + 1). \tag{31}$$

The parameter c is the average connectivity of the lattice. This model can be exactly solved, the main difference with respect to the SK model is that an infinite set of order parameters appear. This makes it more difficult to obtain closed expressions for the thermodynamic properties [12]. Despite the technical difficulties present in this model we expect to obtain the same mean-field chaos exponents. The FC model has a phase transition at a temperature β_c given by

$$1 = (c - 1)\tanh^2(\beta_c). \tag{32}$$

This expression implies that to have a phase transition, we need $c > 2$. To compare this with the results of the 4D case, we have simulated the FC model with $c = 8$, in order to have the same number of nearest neighbours (on average) as the 4D model. The transition temperature is in this case $T_c \simeq 2.76$. We have simulated this model at the critical temperature and below that temperature, at $T = 2.0$. The results for the chaos parameter a are shown in figures 3 and 4. The agreement with the scaling predictions ((25) and (26)) is also fairly good.

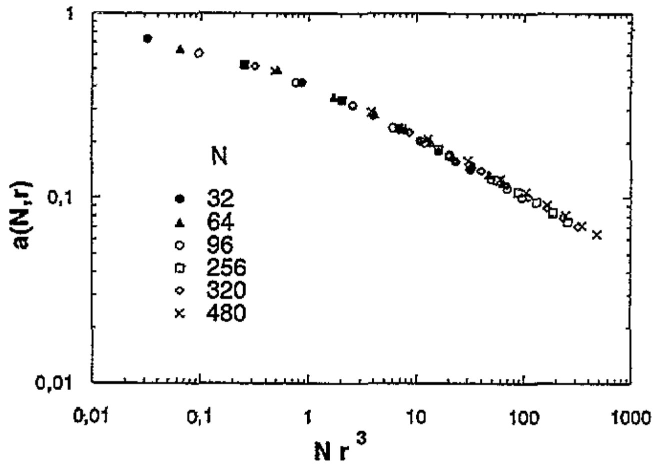

Figure 3. Chaos in the FC model with $c = 8$ at the critical point $T_c \simeq 2.76$.

† One can also consider the case in which the connectivity is fixed and equal to c.

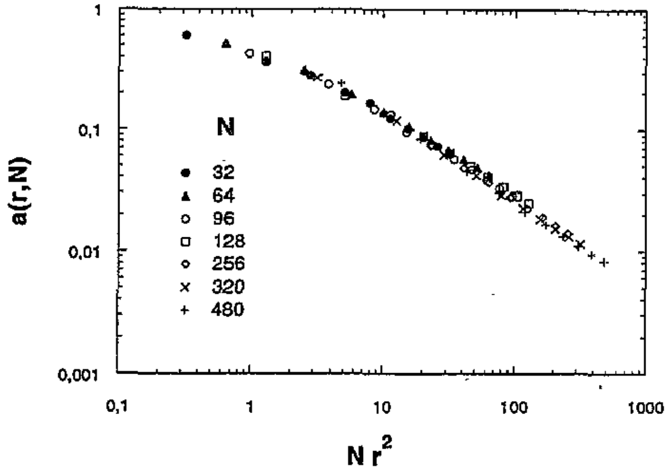

Figure 4. Chaos in the FC model with $c = 8$ at $T = 2.0$ in the spin-glass phase.

4.3. Numerical results in four dimensions

We have also done numerical simulations of the Ising spin-glass model in four dimensions with the purpose of analysing the dimensionality effects on the chaos exponent. We have considered the Ising spin glass at $d = 4$ because it is widely accepted that there is a finite T phase transition in this case†.

We have simulated the model (equation (1)) with a nearest-neighbour interaction, periodic boundary conditions and using a discrete binary distribution of couplings as in equation (31). The model has a transition at $T_c \sim 2.05$ [16]. We have done simulations at $T = T_c$ and $T = 1.7$. The results are shown in figures 5 and 6.

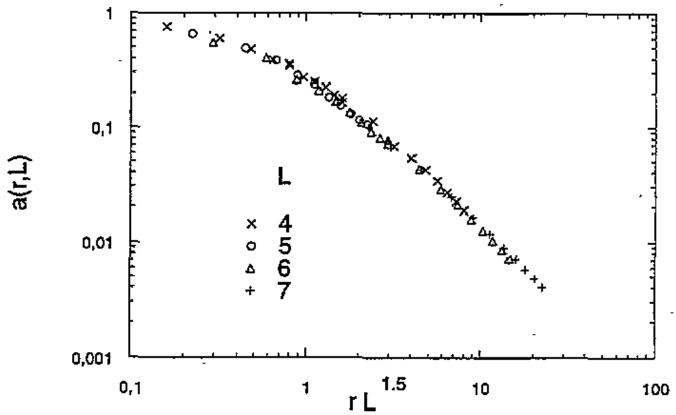

Figure 5. Chaos in the 4D Ising spin glass at the critical temperature $T_c \simeq 2.05$. We obtain $\lambda \sim 2/3$ for the chaos exponent.

At the critical point we obtain a chaos exponent $\lambda \sim 0.65 \pm 0.2$. We have not done a precise determination of the best fit parameter. The values of the error bars have been estimated by looking at the range of parameters which make data collapse in a single

† In the three-dimensional case there is still much controversy on the existence of a finite T transition [5].

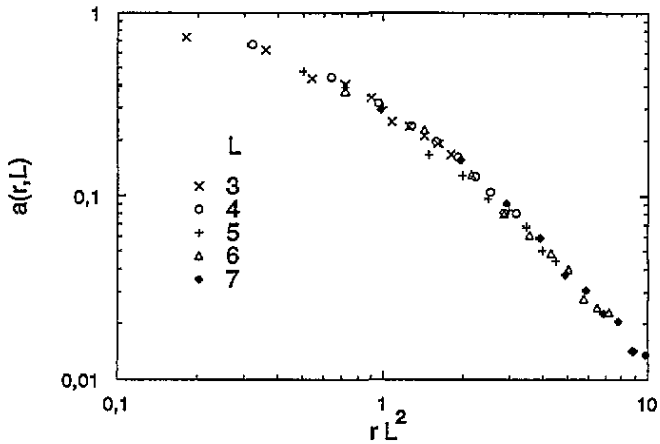

Figure 6. Chaos in the 4D Ising spin glass at $T = 1.7$ in the spin-glass phase. The mean-field chaos exponent $\lambda = \frac{1}{2}$ fits the data very well.

scaling curve. Using equation (8) and the hyperscaling relation $\alpha = 2 - d\nu$ we get $\nu = 0.8 \pm 0.1$. This last value is in agreement with the approximate values of the critical exponents determined for the Ising spin glass in four dimensions (discrete couplings [16] or Gaussian [17]). It would be very interesting to perform extensive simulations to determine whether this exponent is the same for Gaussian couplings. This would suggest that this chaos exponent is universal contrary to what has been observed in the usual critical exponents [18], even though there is not full consensus on this point. It is interesting to speculate on the possibility that the chaos exponents are the relevant ones (in the sense that they are universal) with which to describe the critical behaviour of spin glasses.

The results in figure 6 (at $T = 1.7$, within the spin-glass phase!) show that equation (27) with $\lambda = \frac{1}{2}$ is in pretty good agreement with the data. Our numerical estimate for λ gives $\lambda = 0.5 \pm 0.05$ which is compatible with the mean-field result.

Now we will show that four dimensions is also compatible with some critical dimension for the criticality of chaos. In order to get this result, we will compare the different values of the chaos parameter a for different sizes with the corresponding values of the FC model with $c = 8$. We do the comparison with the FC model, instead of the SK model, because we expect that any logarithmic corrections that are present will be smaller in the FC model than in the SK model. Both are mean-field models even though the FC model resembles the finite d model much more than the SK model does. This fact should be reflected in the nature of the corrections to the universal mean-field behaviour. It is clear that in order to compare the FC model with the four-dimensional model we have to put the system in equivalent points within the phase diagram. We expect the universal function $f(x)$ to depend on the temperature (which is an external parameter) in the following way:

$$a = f(A(T)(L/\xi)Nd). \tag{33}$$

In four dimensions, the scaling function f still depends on the temperature via the universal amplitude $A(T)$. It is reasonable to assume that the dependence of the amplitude $A(T)$ on the temperature enters through the spin-glass order parameter $q(T)$. More concretely, below but close to T_c we expect

$$A(T) \sim q^2(T) \tag{34}$$

because the argument of the scaling function f of equation (33) scales like the singular part of the free energy which in mean-field theory scales like Q_{ab}^2 (see equation (11)). Consequently we have to normalize the adimensional ratio $(L/\xi)^d$ to the corresponding value of the Edwards–Anderson order parameter for that temperature. For $N = 256$ the FC model gives $q(T = 2.0) \simeq 0.11$ and the 4D model at $L = 4$ gives $q(T = 1.5) \simeq 0.25$, the ratio of both numbers being 2.5. Simulation data for both models are shown in figure 7. If one considers the SK model then one observes that data fits well but not so nicely as in the case of the FC model.

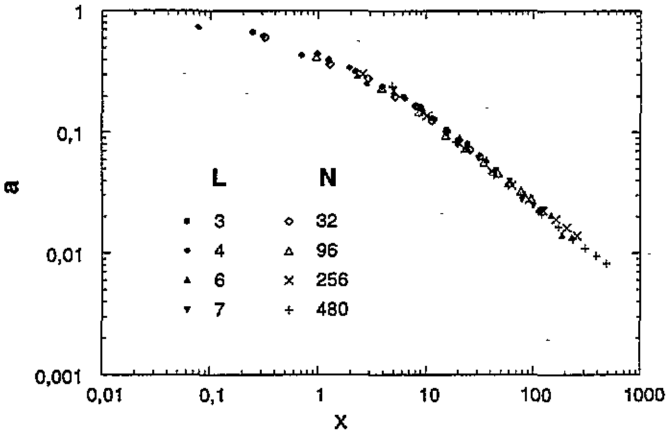

Figure 7. Chaos in the 4D Ising spin glass at $T = 1.7$ compared to the FC model with $c = 8$. The abcisca x corresponds to Nr^2 (with $N = L^4$ in four dimensions). This scaling suggests that four dimensions is the upper critical dimension for the criticality of chaos.

5. Conclusions

We have investigated the sensitivity of spin glasses with respect to the application of a particular static perturbation. In particular, we have studied the nature of the static chaos when a perturbation to the realization of the quenched disorder is applied to the system. This can be done in several ways. In our case we have considered a perturbation which, on average, does not add energy to the system. Due to the self-averaging property we expect that a change in the sign of a finite fraction of the total number of couplings in the system should not change its mean statistical properties (and in particular, its energy). This ensures that the new perturbed system stays at the same point in the phase diagram. The existence of strong chaos for this type of perturbation proves that the reshuffling of the Boltzmann weights of the different states is complete. This differs from the case where the perturbation consists in applying a magnetic field to the system or where its temperature is changed. In these cases extra energy is supplied to the system.

We have solved the mean-field theory and we have extracted the chaos exponent for this type of perturbation. The analytical solution of this problem is very similar to that of chaos against magnetic field perturbations where the chaos correlation length can be exactly computed [3]. This is in contrast to what happens when the temperature is changed. In the last case the system is also chaotic (as recently shown in [8]) but finite-size corrections are very much important.

A finite-size scaling approach to the criticality of chaos shows that $d = 4$ plays the role of an upper critical dimension for the chaos problem. Finite-size scaling studies are very powerful in order to get the chaos exponents. This is because we only need to determine one free parameter to make the data corresponding to different sizes to collapse in a unique scaling function. We have performed numerical simulations of mean-field models which are in agreement with the theory. Simulations in four dimensions are in very good agreement with the fact that 4 plays the role of a critical dimension for the chaos exponent (see figure 7). This is indeed very similar to what happens in the case of magnetic field perturbations.

Finally we would like to point out two possible extensions of this work. First it would be interesting to do dynamical studies of the relaxation of the overlap function against this type of perturbation (as was done for the remanent magnetization after application of a magnetic field). We expect to see aging effects as in the case of magnetic field perturbations. Second, it would be interesting to extend the study of chaos to the metastable states using the TAP formalism. Most probably, similar chaotic properties will be observed in the structure of the metastable states.

Acknowledgments

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References

[1] For general reviews on spin glasses see:
Mézard M, Parisi G and Virasoro M A 1987 *Spin Glass Theory and Beyond* (Singapore: World Scientific)
Fischer K H and Hertz J A 1991 *Spin Glasses* (Cambridge: Cambridge University Press)
Parisi G 1992 *Field Theory, Disorder and Simulations* (Singapore: World Scientific)
Binder K and Young A P 1986 Spin glasses: experimental facts, theoretical concepts and open questions *Rev. Mod. Phys.* **58** 801

[2] McMillan W L 1984 Scaling theory of Ising spin glasses *J. Phys. C: Solid State Phys.* **17** 3179
Bray A J and Moore M A 1985 The nature of the spin-glass phase and finite-size effects *J. Phys. C: Solid State Phys.* **18** L699
Fisher D S and Huse D A 1988 Equilibrium behavior of the spin-glass ordered phase *Phys. Rev.* B **38** 386; 1988 Non-equilibrium dynamics of spin-glasses *Phys. Rev.* B **38** 373

[3] Kondor I 1989 On chaos in spin glasses *J. Phys. A: Math. Gen.* **22** L163

[4] Ritort F 1994 Static chaos and scaling behavior in the spin-glass phase *Phys. Rev.* B **50** 6844

[5] Marinari E, Parisi G and Ritort F 1994 On the 3D Ising spin glass *J. Phys. A: Math. Gen.* **27** 2687 and references therein

[6] Bray A J and Moore M A 1986 *Heidelberg Colloquium in Spin Glasses (Springer Lecture Notes in Physics 275* (Berlin: Springer)

[7] Kawashima N, Hatano N and Suzuki M 1992 Critical behavior of the two-dimensional EA model with a Gaussian bond distribution *J. Phys. A: Math. Gen.* **25** 4985

[8] Franz S and Nifle M N On chaos in mean-field spin glasses *Cond. Mat.* 9412083

[9] Kondor I and Vegso A 1993 Sensitivity of spin glass order to temperature changes *J. Phys. A: Math. Gen.* A **26** L641

[10] Ritort F unpublished results

[11] Cieplak M and Banavar J R 1992 Scaling and phase transitions in random systems *Statistical Physics (Stat. Phys. 18)* (Amsterdam: North-Holland)

[12] Goldschmidt Y Y and De Dominicis C 1990 Replica symmetry breaking in the spin-glass model on lattices with finite connectivity *Phys. Rev.* B **41** 2184 and references therein

[13] Nifle M and Hilhorst H J 1991 *J. Phys. A: Math. Gen.* **24** 2397

[14] Nifle M and Hilhorst H J 1992 New critical-point exponent and new scaling laws for short-ranged Ising spin glasses *Phys. Rev. Lett.* **68** 2992

[15] Sherrington D and Kirkpatrick S 1978 Infinite ranged models of spin glasses *Phys. Rev.* B **17** 4384

[16] Badoni D, Ciria J C, Parisi G, Pech J, Ritort F and Ruiz J J 1993 Numerical evidence of a critical line in the 4D Ising spin glass *Europhys. Lett.* **21** 495

[17] Bhatt R N and Young A P 1988 Numerical studies of Ising spin glasses in two, three and four dimensions *Phys. Rev.* B **37** 5606

[18] Campbell I A 1995 Talk given at Nordita (Copenhaghen) March 1995; received *Preprint*